# Parametric Resonance and Theory of Bragg Waveguides

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Bragg waveguide (1D photonic crystal) consists of a silica core surrounded with several layers of a doped material. Having modified refraction index, this layered cladding forms an interference mirror that confines trapped waveguide modes. Such optical fibers are used for generation, amplification and transmission of coherent infrared beams with large mode area. An important element of Bragg fiber design is the creation of an optimal dielectric permittivity profile providing maximum field attenuation in the quasiperiodic waveguide cladding, and therefore minimum radiation losses. Here, an analogy is obvious with the optimal way of swinging, known to every child.

When rocking a swing, the child is solving a typical problem of optimal control. Mathematically, the process is described by an oscillator equation with varying angular frequency. Its periodic variations can cause parametric resonance. It can be easily noted that the law of varying the efficient pendulum length followed by an experienced "swinger" is far away from the sinusoidal modulation described in standard courses of oscillation theory [1]. By finding the best modulation mode, the child intuitively obtains an oscillating Floquet solution with maximum possible increment. The secret of successful solving the complicated optimization problem lies in the right choice of the independent variable - current oscillation phase.

Electromagnetic field attenuation in the cladding of a Bragg waveguide is similar to the process of a swing slowdown by periodic modulation of pendulum length with the opposite phase. The decisive role of phase synchronism in maximizing the decrement of wave field attenuation manifests itself in the concept of a quarter-wavelength stack, following from the analysis of wave propagation in piece-wise uniform dielectric media [2]. Applied to infrared Bragg waveguides, quarter-wavelength design yields a meander structure with narrow inclusions of an optically denser material, providing small radiation losses even with a little number of dielectric layers [3].

In our work [4], the concept of phase synchronism has been put in a rigorous mathematical form. Consider the equation of anharmonic linear oscillator:

$$u'' + \omega^2(t)\,u = 0 \qquad (1)$$

It has an analytic solution only for a few particular cases $\omega(t)$. The situation radically changes if we consider the oscillation phase as a new independent variable: $\psi(t) = \operatorname{arccot}\left[u'(t)/\omega(t)\,u(t)\right]$ and look for a parametric solution $t = T(\psi), \omega(t) = \Omega(\psi), u(t) = U(\psi)$. In such a way, we obtain a continuum of solutions

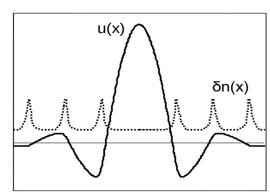
$$\begin{aligned} T(\psi) &= \int\left(\frac{1}{\Omega} - \frac{\dot{\Omega}}{2\Omega^2}\sin 2\psi\right)d\psi, \\ U(\psi) &= \sin\psi\,\exp\left(-\int\frac{\dot{\Omega}}{\Omega}\cos^2\psi\,d\psi\right) \end{aligned} \qquad (2)$$

describing a wide class of anharmonic oscillations. For a periodic $\Omega(\psi) = \Omega(\psi + \pi)$ , function $U(\psi)$ can be exponentially growing, Eqs. (2) being a parametric representation of a particular Floquet solution $u(t) = y(t)\exp(\mu t), \quad y(t + 2\tau) = y(t)$. A marked feature of the phase parameter method is an explicit representation of the oscillation period $\tau$ and increment $\mu$ in terms of arbitrary function $\Omega(\psi) = \bar{\omega}\exp\left[-g(\psi)\right]$:

$$\tau = \frac{2}{\bar{\omega}}\int_0^{\pi} e^{g(\psi)}\sin^2\psi\,d\psi, \quad \mu = \frac{1}{\tau}\int_0^{\pi} g(\psi)\sin 2\psi\,d\psi \quad (3)$$

showing that parametric resonance is determined by a single odd harmonic $b_2$ in the Fourier expansion

$$g(\psi) = a_0 + \sum_{m=1}^{\infty}\left(a_{2m}\cos 2m\psi + b_{2m}\sin 2m\psi\right) \qquad (4)$$




An evident analogy between the oscillator equation (1) and 1D wave equation $u'' + q^2(x)\,u = 0$ allows one to use the obtained solution for the optimization of the refraction index profile in the cladding of a Bragg waveguide. An example of smooth index profile, corresponding to a single harmonic of transversal wave number $q(x) = \sqrt{k^2 n^2(x) - \beta^2}$ - see figure, demonstrates fast field attenuation in the cladding for small values of index variations $\delta n(x)/n_0 \sim 10^{-2}$.

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