Theory and Technology of Wave Vision

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Wave tomography and Wave Vision have many advantages over the already traditional X-ray tomography. This, above all, is almost complete safety for personnel and percipients, as well as the possibility of an unlimited number of repetitions of measurements. In addition, the potential resolving power of a wave tomography is determined by the size of the area of focus achieved, which is related to the wavelength of the radiation used. In the traditional tomography of shadow projections, the resolution is determined by the cross-section of the collimator holes. This means that the use of the wave approach for X-ray can increase the resolution by at least 3 orders of magnitude. Another property of wave tomography - it may be one-sided, for example, the walls of buildings may be probed on one side without going out.

The basis of wave tomography is the possibility of focusing the radiation used at a given point in space. Scanning this point with the investigated space, one can obtain a volumetric resolution of inhomogeneities, which is called a tomogram [1].

Let us explain the formulation of the problem of wave tomography. If $j(\mathbf{r}_1)$ - the distribution of sources of secondary radiation concentrated in a certain volume V_1, then the reflected field created by them at the point of a homogeneous medium is defined as

$$E\left(\mathbf{r}\right)=\iiint\limits_{V_1} j(\mathbf{r}_1)G_0\left(\mathbf{r}_1-\mathbf{r}\right)d^3\mathbf{r}_1. \qquad (1)$$

Это уравнение в свертках. Здесь $G_0(\mathbf{r})$ – функция Грина свободного пространства, т.е. решение уравнения Гельмгольца для свободного пространства. Эта функция представима в виде разложения Вейля по плоским волнам

This equation is convolution. Here $G_0(\mathbf{r})$ is the Green's function of free space, it is solution of the Helmholtz equation for free space. This function can be represented in the form of a Weyl expansion in plane waves

$$G_0(\mathbf{r})=\exp(ik|\mathbf{r}|)/4\pi|\mathbf{r}|=\frac{i}{(2\pi)^2}\iint\frac{\exp\left\{i\left(\boldsymbol{\kappa}_\perp\boldsymbol{\rho}+\kappa_z z\right)\right\}}{2\kappa_z}\left(d^2\boldsymbol{\kappa}_\perp\right).$$

Here $\kappa_z=\sqrt{k^2-\boldsymbol{\kappa}_\perp^2}$ is the longitudinal projection of the wave vector. As a result, from (1) we can write for the spectrum of spatial frequencies of the recorded wave field

$$\mathrm{E}(\boldsymbol{\kappa}_\perp,z)=\iint\exp\left\{-i\boldsymbol{\kappa}_\perp\boldsymbol{\rho}\right\}\mathrm{E}(\boldsymbol{\rho},z)\left(d^2\boldsymbol{\rho}_\perp\right)=\frac{i}{2\kappa_z}\exp\left\{-i\kappa_z z\right\}j\left(\boldsymbol{\kappa}\right), \qquad (2)$$

here

$$j\left(\boldsymbol{\kappa}\right)\equiv\iiint\limits_{V_1} j(\mathbf{r}_1)\exp(\mathrm{i}\,\boldsymbol{\kappa}\mathbf{r}_1)d^3\mathbf{r}_1 \ -$$

is a three-dimensional Fourier transform from the spatial distribution of sources-radiation currents.

Expression (2) allows us to write down that

$$j\left(\boldsymbol{\kappa}\right)=-2i\kappa_z\exp\left\{i\kappa_z z\right\}\mathrm{E}(\boldsymbol{\kappa}_\perp,z).$$

The resulting representation is a 3D tomography solution. It is sufficient only to perform a 3-D inverse Fourier transform. This solution is essentially the basic idea of the well-known Stolt method [2].

The report gives many concrete examples of Wave Vision and wave tomography.

Reference

[1]. Yakubov V. P., Shipilov S. E., Sukhanov D. Ya., Klokov A. V. Wave tomography / ed. by V. P. Yakubov. – Tomsk: Scientific Technology Publishing House, 2017. – 248 p.
[2]. Stolt R.H. Migration by Fourier transform // Geophysics. – 1978. – V. 43. – No. 1. – P. 23−48.